$$\frac{1}{3}\int \cos 3x \cdot 3\, dx$$

$$u = 3x$$

$$du = 3\, dx$$

$$\frac{1}{3}\int \cos u \, du$$

$$\int \cos \boxed{3x}\, dx$$

$$\frac{1}{3}\sin 3x + C$$

$$\int \cos 4x \, dx = \frac{1}{4}\sin 4x + C$$

$$\int \sin 5x \, dx = -\frac{1}{5}\cos 5x + C$$

$$\int \sec^2 2x \, dx = \frac{1}{2}\tan 2x + C$$

$$\frac{1}{3}\int 3e^{3x}\,dx \qquad u = 3x$$

$$du = 3\,dx$$

$$\frac{1}{3}\int e^{u}\,du = \frac{1}{3}e^{u}$$

$$= \frac{1}{3}e^{3x} + c$$

$$\int e^{4x}\,dx = \frac{1}{4}e^{4x} + C$$

$$\int e^{5x}\,dx = \frac{1}{5}e^{5x} + C$$

$$\frac{1}{2}\int \underline{2x} \sin \underline{x^2}\, \underline{dx} \qquad u = \underline{x^2}$$

$$du = \underline{2x\,dx}$$

$$\frac{1}{2}\int \sin u\, du$$

$$= \frac{1}{2}(-\cos u) = -\frac{1}{2}\cos x^2 + C$$

What if $\int x \sin x\, dx$

Integration by Parts

(uv substitution)

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Why? $f(x) = uv$

$$(uv)' = u'v + v'u$$

$$\int (uv)' = \int u'v + \int v'u$$

$$uv = \int v\,du + \int u\,dv$$

$$uv - \int v\,du = \int u\,dv$$

$$\int u\,dv = uv - \int v\,du$$

$$\underline{x}\ \underline{\sin x\,dx}$$

$$\int x \sin x\,dx \qquad u = x \qquad dv = \sin x\,dx$$

$$du = 1\,dx \qquad v = -\cos x$$

$$= -x\cos x - \int -\cos x\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

$$\int x \cos 3x \, dx$$

$u = x \qquad dV = \cos 3x \, dx$

$du = 1\,dx \qquad v = \frac{1}{3} \sin 3x$

$$= \frac{1}{3} x \sin 3x - \int \frac{1}{3} \sin 3x \, dx$$

$$= \frac{1}{3} x \sin 3x - \frac{1}{3} \cdot \frac{1}{3} (-\cos 3x)$$

$$= \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x + C$$

$$\int x e^{3x}\,dx \qquad u = x \quad dv = e^{3x}\,dx$$

$$du = 1\,dx \quad v = \frac{1}{3}e^{3x}$$

$$= \frac{1}{3}e^{3x}x - \int \frac{1}{3}e^{3x}\,dx$$

$$= \frac{1}{3}e^{3x}x - \frac{1}{3}\cdot\frac{1}{3}e^{3x}$$

$$= \frac{1}{3}e^{3x}x - \frac{1}{9}e^{3x} + C$$

$$\int x^2 e^x \, dx$$

$u = x^2 \quad dv = e^x \, dx$

$du = 2x \, dx \quad v = e^x$

$$= x^2 e^x - \int 2x e^x \, dx$$

$$= x^2 e^x - 2 \int x e^x \, dx$$

$u = x \quad dv = e^x \, dx$

$du = 1 \, dx \quad v = e^x$

$$= x^2 e^x - 2\left(x e^x - \int e^x \, dx \right)$$

$$= x^2 e^x - 2(x e^x - e^x) = x^2 e^x - 2x e^x + 2e^x + C$$

$$\int x^3 \sin 2x \, dx$$

tic-tac-toe

	u	dV
$+$	x^3	$\sin 2x$
$-$	$3x^2$	$-\frac{1}{2}\cos 2x$
$+$	$6x$	$-\frac{1}{4}\sin 2x$
$-$	6	$\frac{1}{8}\cos 2x$
$+$	0	$\frac{1}{16}\sin 2x$

$$-\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2\sin 2x + \frac{3}{4}x\cos 2x - \frac{3}{8}\sin 2x + C$$

$$\int x^3 e^{2x}\,dx$$

	u	dv
$+$	x^3	e^{2x}
$-$	$3x^2$	$\frac{1}{2}e^{2x}$
$+$	$6x$	$\frac{1}{4}e^{2x}$
$-$	6	$\frac{1}{8}e^{2x}$
$+$	0	$\frac{1}{16}e^{2x}$

$$\frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x} + C$$

$$\int t^{\frac{1}{2}} \ln t \, dt \qquad u = \ln t \quad dv = t^{\frac{1}{2}} dt$$

$$du = \frac{1}{t} dt \quad v = \frac{2}{3} t^{3/2}$$

$$\frac{2}{3} t^{3/2} \ln t - \int \frac{2}{3} t^{3/2} \cdot \frac{1}{t} \, dt$$

$$\frac{2}{3} t^{3/2} \ln t - \frac{2}{3} \int t^{1/2} \, dt$$

$$- \frac{2}{3} \cdot \frac{2}{3} t^{3/2}$$

$$= \frac{2}{3} t^{3/2} \ln t - \frac{4}{9} t^{3/2} + C$$

$$\int \ln x \, dx$$

$u = \ln x \quad dv = 1\,dx$

$du = \frac{1}{x}dx \quad v = x$

$$x \ln x - \int x \cdot \frac{1}{x} dx$$

$$x \ln x - \int 1 \, dx$$

$$x \ln x - x + C$$

$$\int x5^x\,dx \qquad u = x \quad dv = 5^x\,dx$$

$$du = 1\,dx \quad v = \frac{5^x}{\ln 5}$$

$$\frac{x5^x}{\ln 5} - \int \frac{5^x}{\ln 5}\,dx$$

$$\frac{x5^x}{\ln 5} - \frac{1}{\ln 5}\int 5^x\,dx$$

$$\frac{x5^x}{\ln 5} - \frac{1}{\ln 5}\cdot\frac{5^x}{\ln 5} = \frac{x5^x}{\ln 5} - \frac{5^x}{(\ln 5)^2} + C$$

$$\int \sin^{-1} x \, dx$$

$u = \sin^{-1} x \qquad dv = 1\,dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \qquad v = x$

$$x \sin^{-1} x - \frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$$

$u = 1 - x^2$

$du = -2x\,dx$

$$x \sin^{-1} x + \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$$

$$x \sin^{-1} x + \frac{1}{2}\int u^{-1/2}$$

$$\text{"} \quad + \frac{1}{2}\cdot 2u^{1/2} = x \sin^{-1} x + (1-x^2)^{1/2} + C$$