

8-3

Objective

After studying this lesson, you should be able to:

- solve systems of equations by using the elimination method with addition or subtraction.

APPLICATION Entertainment

Disney cartoons are animated using an expensive computer process that makes the action flow smoothly and seem lifelike. In 1994, Disney's animated feature *The Lion King* was the top-grossing film of the year, making an estimated \$300.4 million at the box office.

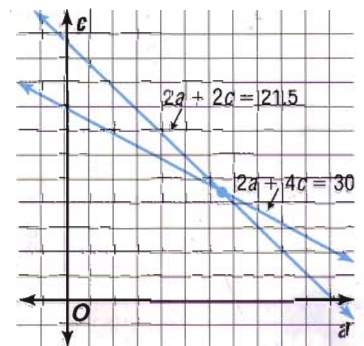


On a Saturday afternoon, the Johnson and Olivera families decided to go see *The Lion King* together. The Johnson family, two adults and four children, can afford to spend \$30 from their entertainment budget this weekend for the movie tickets, while the Olivera family, two adults and two children, can afford to spend \$21.50. Different theaters around town charge different amounts for adult and child tickets. What price can the Johnsons and Oliveras afford to pay for each adult and each child?

Let a represent the ticket price for one adult, and let c represent the ticket price for one child. Then the information in this problem can be represented by the following system of equations.

$$\begin{aligned} 2a + 4c &= 30 \\ 2a + 2c &= 21.5 \end{aligned}$$

From the graph at the right, an estimated solution is (\$7, \$4). To get an exact solution, solve algebraically. You could solve this system by first solving either of the equations for a or c and then using substitution.



However, a simpler method of solution is to subtract one equation from the other since the coefficients of the variable a are the same. This method is called **elimination** because the subtraction eliminates one of the variables. First, write the equations in column form and subtract.

$$\begin{array}{r} 2a + 4c = 30 \\ (-) 2a + 2c = 21.5 \\ \hline \end{array} \quad \xrightarrow{\text{Multiply by } -1.} \quad \begin{array}{r} 2a + 4c = 30 \\ (+) -2a - 2c = -21.5 \\ \hline 2c = 8.5 \\ c = 4.25 \end{array}$$

Then, substitute 4.25 for c in either equation and find the value of a .

$$\begin{aligned} 2a + 2c &= 21.5 \\ 2a + 2(4.25) &= 21.5 && \text{Substitute 4.25 for } c. \\ 2a + 8.5 &= 21.5 \\ 2a &= 13 \\ a &= 6.5 \end{aligned}$$

Is (6.5, 4.25) a solution of the system?

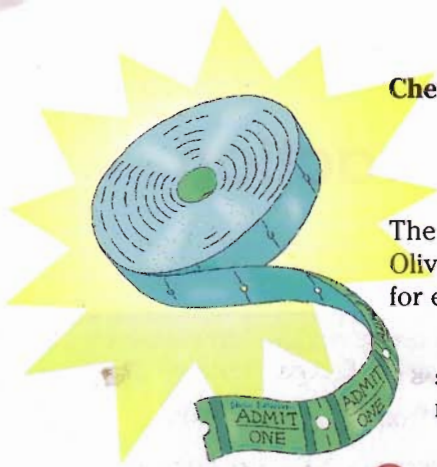
fabulous FIRSTS



Elmer Simms Campbell
(1906–1971)

Elmer Simms Campbell was the first African-American cartoonist to work for national publications. He contributed cartoons and other art work to *Esquire*, *Cosmopolitan*, and *Redbook*, as well as syndicated features in 145 newspapers.

Recall that subtraction is the same as adding the opposite.



Check: $2a + 4c = 30$
 $2(6.5) + 4(4.25) \stackrel{?}{=} 30$
 $30 = 30 \quad \checkmark$

$2a + 2c = 21.5$
 $2(6.5) + 2(4.25) \stackrel{?}{=} 21.5$
 $21.5 = 21.5 \quad \checkmark$

The solution of this system of equations is (6.5, 4.25). Thus, the Johnsons and Oliveras should look for a theater that charges \$6.50 for each adult and \$4.25 for each child.

In some systems of equations, the coefficients of terms containing the same variable are additive inverses. For these systems, the elimination method can be applied by adding the equations.

Example 1 Use elimination to solve the system of equations.

$$\begin{aligned} 3x - 2y &= 4 \\ 4x + 2y &= 10 \end{aligned}$$

Since the coefficients of the y -terms, -2 and 2 , are additive inverses, you can solve the system by adding the equations.

$$\begin{array}{r} 3x - 2y = 4 \quad \text{Write the equations in column form and add.} \\ (+) 4x + 2y = 10 \quad \text{Notice that the variable } y \text{ is eliminated.} \\ \hline 7x \quad = 14 \\ x = 2 \end{array}$$

Now substitute 2 for x in either equation to find the value of y .

$$\begin{aligned} 3x - 2y &= 4 \\ 3(2) - 2y &= 4 \\ 6 - 2y &= 4 \\ -2y &= -2 \\ y &= 1 \end{aligned}$$

The solution of this system is (2, 1). *Check this result.*

Use subtraction to solve a system of two linear equations whenever one of the variables has the same coefficient in both equations.

Example 2 The sum of two numbers is 18. The sum of the greater number and twice the smaller number is 25. Find the numbers.

INTEGRATION
Number Theory

Let x = the greater number and let y = the lesser number. Since the sum of the numbers is 18, one equation is $x + y = 18$. Since the sum of the greater number and twice the smaller number is 25, the other equation is $x + 2y = 25$. Use elimination to solve this system.

$$\begin{array}{r} x + y = 18 \\ (-) x + 2y = 25 \quad \text{Since the coefficients of the } x \text{ terms are} \\ \hline -y = -7 \quad \text{the same, use elimination by subtraction.} \\ y = 7 \end{array}$$

Find x by substituting 7 for y in one of the equations.

$$\begin{aligned} x + y &= 18 \\ x + 7 &= 18 \quad \text{Substitute 7 for } y. \\ x &= 11 \quad \text{Solve for } x. \end{aligned}$$

The solution is (11, 7), which means that the numbers are 7 and 11. *Check this result.*

Several software packages can be used to help you solve systems of equations.



EXPLORATION

GRAPHING SOFTWARE

The *Mathematics Exploration Toolkit (MET)* can be used to graph and solve systems of equations. Use the following CALC commands.

CLEAR F (clr f) Removes previous graphs from the graphing window.

GRAPH (gra) Graphs the most recent equation in the expression window.

SCALE (sca) Sets limits on the x - and y -axes.

To set up the graphing window, enter clr f. Then enter sca 10. This sets limits on the axes at -10 to 10 for x and y . Since only two points are needed to graph a line, use the command gra 2.

Your Turn

- Check the solutions of the examples using *MET*.
- Graph the system $3x + 2y = 7$ and $5x + 2y = 17$ and find the solution using *MET*.
- Describe the difference between solving a system of equations using graphing software and using a graphing calculator. Which do you prefer, and why?

Example



APPLICATION

Testing

- 3** Lina is preparing to take the Scholastic Assessment Test (SAT). She has been taking practice tests for a year, and her scores are steadily improving. She always scores about 150 points higher on the math test than on the verbal test. She needs a combined score of 1270 to get into the college she has chosen. If she assumes she will still have that 150-point difference between the two tests, how high does she need to score on each part?

Let m represent Lina's math score. Let v represent Lina's verbal score. Since the sum of her scores is 1270, one equation is $m + v = 1270$. Since the difference of her scores is 150, another equation is $m - v = 150$. Use elimination to solve this system.

$$\begin{array}{r} m + v = 1270 \\ (+) m - v = 150 \\ \hline 2m = 1420 \\ m = 710 \end{array}$$

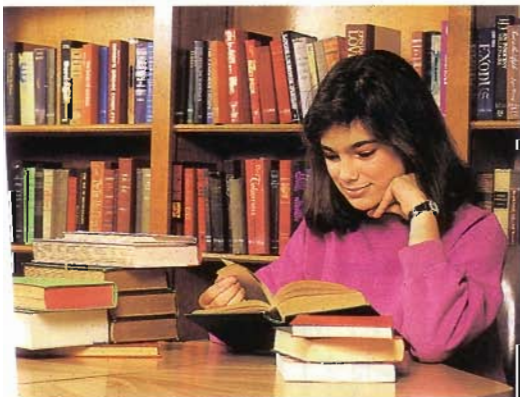
Since the coefficients of the v term are additive inverses, use elimination by addition.

Find v by substituting 710 for m in one of the equations.

$$\begin{array}{r} m + v = 1270 \\ 710 + v = 1270 \\ v = 560 \end{array}$$

Substitute 710 for m .

The solution is $(710, 560)$, which means that Lina must score 710 on the math portion and 560 on the verbal portion of the SAT.



CHECK FOR UNDERSTANDING

Communicating Mathematics

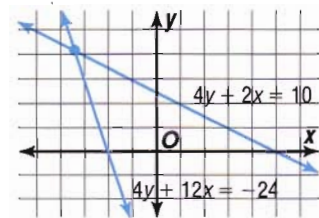
Study the lesson. Then complete the following.

1. **Explain** when it is easier to solve a system of equations in each way.
 - a. by elimination using subtraction
 - b. by elimination using addition
2. **a. State** the result when you add $3x - 8y = 29$ and $-3x + 8y = 16$. What does this result tell you about the system of equations?

b. What does this result tell you about the graph of the system?
3. **You Decide** Maribela says that a system of equations has no solution if both variables are eliminated by addition or subtraction. Devin argues that there may be an infinite number of solutions. Who is correct? Explain your answer.

Guided Practice

4. Refer to the graph at the right.
 - a. Estimate the solution of the system.
 - b. Use elimination to find the exact solution.



State whether addition, subtraction, or substitution would be most convenient to solve each system of equations. Then solve the system.

- | | | |
|------------------|------------------|-------------------|
| 5. $3x - 5y = 3$ | 6. $3x + 2y = 7$ | 7. $-4m + 2n = 6$ |
| $4x + 5y = 4$ | $y = 4x - 2$ | $-4m + n = 8$ |
| 8. $8a + b = 1$ | 9. $3x + y = 7$ | 10. $2b + 4c = 8$ |
| $8a - 3b = 3$ | $2x + 5y = 22$ | $c - 2 = b$ |

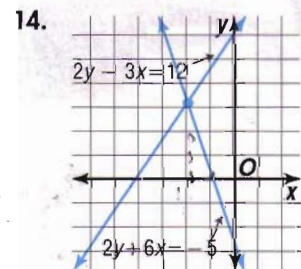
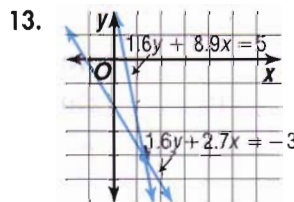
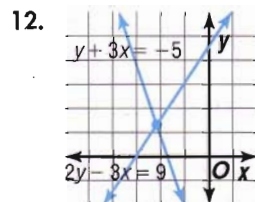
11. **Statistics** The mean of two numbers is 28. Find the numbers if three times one of the numbers equals half the other number.

EXERCISES

Practice

For Exercises 12–14,

- a. estimate the solution of each system of linear equations, and
- b. use elimination to find the exact solution of each system.



State whether addition, subtraction, or substitution would be most convenient to solve each system of equations. Then solve the system.

- | | | |
|---|---|---|
| 15. $x + y = 8$
$x - y = 4$ | 16. $2r + s = 5$
$r - s = 1$ | 17. $x - 3y = 7$
$x + 2y = 2$ |
| 18. $3x + y = 5$
$2x + y = 10$ | 19. $5s + 2t = 6$
$9s + 2t = 22$ | 20. $4x - 3y = 12$
$4x + 3y = 24$ |
| 21. $2x + 3y = 13$
$x - 3y = 2$ | 22. $2m - 5n = -6$
$2m - 7n = -14$ | 23. $x - 2y = 7$
$-3x + 6y = -21$ |
| 24. $3r - 5s = -35$
$2r - 5s = -30$ | 25. $13a + 5b = -11$
$13a + 11b = 7$ | 26. $a - 2b - 5 = 0$
$3a - 2b - 9 = 0$ |
| 27. $4x = 7 - 5y$
$8x = 9 - 5y$ | 28. $\frac{2}{3}x + y = 7$
$\frac{10}{3}x + 5y = 11$ | 29. $\frac{3}{5}c - \frac{1}{5}d = 9$
$\frac{7}{5}c + \frac{1}{5}d = 11$ |
| 30. $0.6m - 0.2n = 0.9$
$0.3m = 0.45 - 0.1n$ | 31. $1.44x - 3.24y = -5.58$
$1.08x + 3.24y = 9.99$ | 32. $7.2m + 4.5n = 129.06$
$7.2m + 6.7n = 136.54$ |



Use a system of equations and elimination to solve each problem.

- Find two numbers whose sum is 64 and whose difference is 42.
- Find two numbers whose sum is 18 and whose difference is 22.
- Twice one number added to another number is 18. Four times the first number minus the other number is 12. Find the numbers.
- If $x + y = 11$ and $x - y = 5$, what does xy equal?

Use elimination twice to solve each system of equations. Write the solution as an ordered triple of the form (x, y, z) .

- | | | |
|--|--|---|
| 37. $x + y = 5$
$y + z = 10$
$x + z = 9$ | 38. $2x + y + z = 13$
$x - y + 2z = 8$
$4x - 3z = 7$ | 39. $x + 2z = 2$
$y + 3z = 9$
$12x - y + 7z = 99$ |
|--|--|---|

Critical Thinking

- The graphs of $Ax + By = 7$ and $Ax - By = 9$ intersect at $(4, -1)$. Find A and B .

Applications and Problem Solving

- On-Line Entertainment** On June 27, 1994, Aerosmith became the first major rock band to release a song distributed exclusively in the U.S. through a computer on-line service. Users of the commercial service, CompuServe, were able to download the Aerosmith song *Head First* for free.



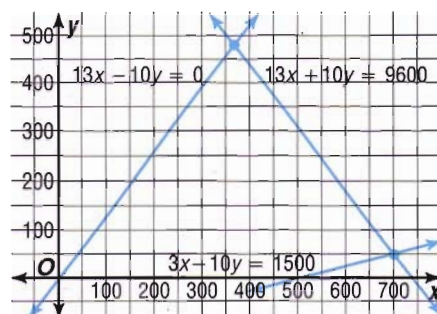
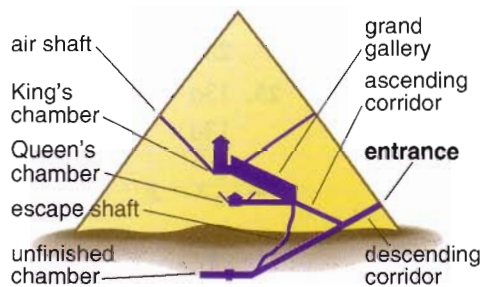
However, it took a long time to download the song, which itself lasted only 3 minutes, 14 seconds, because of the high-fidelity sound. José and Ling share a personal computer, and one evening they each downloaded the song without realizing that the other had done it. Ling also wasted 18 minutes because he typed the wrong word and had to start over. At the end of the month, the bill from CompuServe said they had used a total of 2.6 hours of time that evening. How long did it take to download the song each time? (*Hint:* 18 minutes = 0.3 hour.)

F Y I

The Great Pyramid of King Khufu, built around 2600 B.C., has only one entrance that is nearly impossible to find. Centuries later, however, robbers stole everything from the pyramid, including the mummy of the king.

42. **World Cultures** The ancient Egyptians believed that the pharaohs lived forever after death in their houses of eternity, the pyramids. Suppose the side of the pyramid containing the entrance is represented by the line $13x + 10y = 9600$, the opposite side of the pyramid, by the line $13x - 10y = 0$, and the descending corridor leading to the entrance, by the line $3x - 10y = 1500$, where x is the distance in feet and y is the height in feet.

- Find the coordinates of the entrance.
- Find the height of the pyramid.



Mixed Review

43. **Chemistry** MX Labs needs to make 500 gallons of a 34% acid solution. The only solutions available are 25% acid and 50% acid. How many gallons of each solution should be mixed to make the 34% solution? Write and solve a system of equations by using substitution. (Lesson 8-2)
44. Solve $5 - 8h \leq 9$. (Lesson 7-1)
45. Determine the slope of the line that passes through the points at $(2, -9)$ and $(-1, 0)$. (Lesson 6-1)
46. Graph $6x - \frac{1}{2}y = -10$. (Lesson 5-4)
47. Solve $-2(3t + 1) = 5$. (Lesson 3-3)
48. Find an approximation to the nearest hundredth for $\sqrt{15}$. (Lesson 2-8)
49. Name the property illustrated by the following statement. (Lesson 1-6)
If $6 = 2a$ and $a = 3$, then $6 = 2 \cdot 3$.

SELF TEST

Graph each system of equations. Then determine if the system has *one solution*, *no solution*, or *infinitely many solutions*. If the system has one solution, name it. (Lesson 8-1)

1. $x - y = 3$
 $3x + y = 1$

2. $2x - 3y = 7$
 $3y = 7 + 2x$

3. $4x + y = 12$
 $x = 3 - \frac{1}{4}y$

Use substitution to solve each system of equations. (Lesson 8-2)

4. $y = 5x$
 $x + 2y = 22$

5. $2y - x = -5$
 $y - 3x = 20$

6. $3x + 2y = 18$
 $x + \frac{8}{3}y = 12$

Use elimination to solve each system of equations. (Lesson 8-3)

7. $x - y = -5$
 $x + y = 25$

8. $3x + 5y = 14$
 $2x - 5y = 1$

9. $5x + 4y = 12$
 $3x + 4y = 4$

10. **Recreation** At a recreation and sports facility, 3 members and 3 nonmembers pay a total of \$180 to take an aerobics class. A group of 5 members and 3 nonmembers pay \$210 to take the same class. How much does it cost members and nonmembers to take an aerobics class? (Lesson 8-3)